$$A\Phi_t + zB\Phi_{zt} = z^2 f_1(t, z, \Phi, \Phi_z, \Phi_t, \Phi_{tt}, \Phi_{zt}, \Phi_{zz}) + \qquad (4.9)$$
$$zf_2(z, t, \Phi, \Phi_z, \Phi_{tt}) + f_3(z, t, \Phi)$$

$$\Phi + Cz\Phi_z + z^2\Phi_{zz} = f_4(z, t, z\Phi_t, z^2\Phi_{zt}) \quad \text{for} \quad t = \eta(z)$$

where A, B and C are positive constants and the functions f_1, f_2, f_3 and f_4 are analytic in all variables. By the theorem proved in [5] the problem (4.9) has a unique analytic solution and the series (1.5) as well as the series for the second order derivatives obtained from (1.5) converge in some neighborhood of the point (0, 0).

REFERENCES

1. Sidorov, A. F., On disruption of potential gas flows adjacent to the region of rest. PMM Vol. 35, № 3, 1971.

2. Sidorov, A. F., Method of solution of certain boundary value problems for nonlinear hyperbolic equations and propagation of weak shock waves. PMM Vol. 36, №3, 1972.

3. Zubov, E. N. and Sidorov, A. F., On solution of a boundary value problem for an unsteady three-dimensional gas flow and on the propagation of weak shock waves. Coll. : Numerical Methods in the Mechanics of Continuous Media. Novosibirsk, VTs SO Akad. Nauk SSSR, Vol. 3, № 3, 1972.

4. Landau, L. D. and Lifshits, E. M., Mechanics of Continuous Media, Moscow, Gostekhizdat, 1954.

5. Bautin, S. P., Analytic solutions of the problem of motion of a piston. Coll. : Numerical Methods in the Mechanics of Continuous Media, Novosibirsk, VTs SO Acad. Nauk SSSR, Vol. 4, № 1, 1973.

Translated by L. K.

ON FORCES DUE TO BARNETT STRESSES ACTING ON BODIES IN A GAS

V. S. GALKIN and O. G. FRIDLENDER
(Moscow)

Proof is given of certain statements about forces acting on uniformly heated bodies in a gas. It is shown that bodies heated to different temperatures repel each other, while a heated and a cooled body are mutually attracted. A new form of thermophoresis is indicated. These phenomena are the result of Barnett thermal stresses. The existence of similar effects induced by concentration stresses in gas mixtures is established.

1. Fundamental relationships. When defining slow (characteristic Reynolds number $R \approx 1$ and Mach number $M \ll 1$) flows of gas in a substantially nonuniform temperature field, i.e. whose characteristic relative temperature differentials $\tau_* \approx 1$, it is necessary to take into consideration Barnett thermal stresses

$$p_{ij}^{(T)} = \alpha_1 \left[\frac{\partial^2 \Omega}{\partial x_i \partial x_j} \right] + \alpha_2 \left[\frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_j} \right] \tag{1.1}$$

$$\Omega = \int \eta \, dT, \ [A_{ij}] = \frac{1}{2} (A_{ij} + A_{ji}) - \frac{1}{3} \delta_{ij} A_{kk}, \quad \alpha_{1,2} = \alpha_{1,2}(T)$$

since these are of the fundamental order of magnitude [1 − 4]. Formula (1.1) and the equations of conservation for such flows are defined below in the following dimensionless variables: temperature, thermal conductivity and viscosity coefficients, density, Cartesian coordinates, thermal stresses, velocity, pressure, the force acting on the body, and the moment of forces, respectively, by

$$T_* T, \ \eta_* \eta(T), \quad \mu_* \mu(T), \quad \rho_* \rho \tag{1.2}$$

$$L(x_1, \ x_2, \ x_3) \equiv L(x, \ y, \ z), \quad \frac{\mu_*^2}{\rho_* L^2} p_{ij}^{(T)}, \quad \frac{\mu_*}{\rho_* L} \mathbf{v}$$

$$1 + \frac{\mu_*}{\rho_* L} \left(\frac{k}{m} T_* \right)^{-1/2} p, \quad \frac{\mu_*^2}{\rho_*} \mathbf{F}, \quad \frac{\mu_*^2 L}{\rho_*} \mathbf{M}$$

where L is a characteristic dimension and k / m is the gas constant. Unless otherwise stated, the characteristic values are assumed to be equal to the corresponding parameters of gas in the unperturbed stream $(T_* = T_\infty$, etc.). The ratio of specific heats and the Prandtl number for $T = 1$ are denoted, respectively, by $\varkappa$ and P.

Eliminating ρ as in [2], by using the equation of state $\rho = 1 / T$, we reduce the equations of continuity, energy and momentum for stationary conditions and in the absence of external forces to the form

$$\nabla \mathbf{v} = \mathbf{v} \cdot \nabla \ln T \tag{1.3}$$

$$E \mathbf{v} \cdot \nabla \ln T = \Delta \Omega, \qquad E = \frac{5}{2} (\varkappa - 1) P / \varkappa \tag{1.4}$$

$$T^{-1} (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla \Pi = \tag{1.5}$$
$$\Pi^{(1)} + Y_T (\nabla T)^2 \nabla T + E (\alpha_1^{\cdot} - \alpha_2 / \eta) (\mathbf{v} \cdot \nabla \ln T) \nabla T$$

$$\Pi_i^{(1)} = \frac{\partial}{\partial x_j} \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \qquad A^{\cdot} = \frac{dA}{dT}$$

$$\Pi = p + X_T (\nabla T)^2 + \frac{2}{3} (E \alpha_1 + \mu) (\mathbf{v} \cdot \nabla \ln T) \tag{1.6}$$

$$X_T = \frac{1}{2} \alpha_1^{\cdot} \eta + \frac{1}{6} \alpha_2, \quad Y_T = \frac{1}{2} (\alpha_1^{\cdot\cdot} \eta > \alpha_2^{\cdot} > \alpha_1^{\cdot} \eta^{\cdot} + 2 \alpha_2 \eta^{\cdot} / \eta) \tag{1.7}$$

If $\mu = \eta = T^s$, then [2]

$$\alpha_1 = \omega_1 T^s, \quad \alpha_2 = - \omega_3 T^{2s-1}, \quad \omega_1 > 0, \quad \omega_3 > 0 \tag{1.8}$$

$$X_T = \frac{1}{2} (\omega_1 s - \omega_3 / 3) T^{2s-1}, \quad Y_T = -\frac{1}{2} (\omega_1 s + \omega_3) T^{2s-2}$$

The structure of Barnett coefficients for this particular case is known [5]. In the first approximation by Sonin's polynomials $\omega_1 = 3$ and $\omega_3 = 0$, and this is exact in the case of Maxwellian molecules $(s = 1)$. This approximation yields very good results for μ and η of a simple gas, it is, however, inadequate for Barnett coefficients; in the case of molecules (elastic spheres) $(s = \frac{1}{2})$ it is necessary to resort to four approximations by Sonin's polynomials, which yield $\omega_1 = 2.418$ and $\omega_3 = 0.990$. For $\tau_* \ll 1$ the rate of thermal-stress convection is $v \sim Y_T \tau_*^3$ [2]. For elastic spheres the ratio of Y_T values calculated in the first and fourth approximations by Sonin's polynomials is

$$Y_T^{(1)} / Y_T^{(4)} = {}^3/_2 / ({}^1/_2 \cdot 2.418 + 0.990) = 0.68$$

Hence the first approximation by Sonin's polynomials and the equivalent results obtained by Grad's method may lead to considerable errors, particularly in the case of gas mixtures.

Boundary conditions for system $(1.3) - (1.7)$ differ from those usually applied for solving the Navier-Stokes equations only by the condition for the tangent velocity component along the body $v_\tau = \beta\mu\partial T_w / \partial x_\tau$, where for given gas and material of the wall the coefficient $\beta = \text{const}$ and v_τ is called the thermal creep rate.

The case of the "uniformly heated" body in which the temperature at its surface is everywhere the same $(T_w = \text{const})$ is primarily considered here. In this case at the surface of the body $\mathbf{v} = 0, \ \Delta\mathbf{v} = 0, \ T = T_w$, and

$$\Delta\Omega = 0 \tag{1.9}$$

and the total stress tensor along S, i.e. the tensor of local stresses, is

$$P_{ij} = p\delta_{ij} - 2\mu_w\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \alpha_{1w}\frac{\partial^2\Omega}{\partial x_i\partial x_j} + \alpha_{2w}\left[\frac{\partial T}{\partial x_i}\frac{\partial T}{\partial x_j}\right] \tag{1.10}$$

where $\mu_w = \mu(T_w)$ etc.

The integrals over S with respect to P_{ij} yield $\mathbf{F}$ and $\mathbf{M}$. We denote the related integrals of local thermal stresses, i.e. of the last two terms in formula (1.10), by $\mathbf{F}^{(T)}$ and $\mathbf{M}^{(T)}$, the outward normal by $\mathbf{n}$, the radius vector by $\mathbf{r}$, and the unit vector along an arbitrarily chosen x-axis by $\mathbf{e}_x$.

Lemma 1. When $T_w = \text{const}$, we have

$$F_x^{(T)} = -\frac{2}{3}\alpha_{2w}\int(\nabla T)^2(\mathbf{n}\cdot\mathbf{e}_x)\,dS \tag{1.11}$$

$$M_x^{(T)} = -\frac{2}{3}\alpha_{2w}\int(\nabla T)^2[y(\mathbf{n}\cdot\mathbf{e}_z) - z(\mathbf{n}\cdot\mathbf{e}_y)]\,dS \tag{1.12}$$

In fact

$$F_x^{(T)} = -\int_S p_{kx}^{(T)}(\mathbf{n}\cdot\mathbf{e}_k)\,dS = \frac{1}{3}\alpha_{2w}\int(\nabla T)^2(\mathbf{n}\cdot\mathbf{e}_x)\,dS - \tag{1.13}$$
$$\alpha_{1w}I_1 - \alpha_{2w}I_2$$

Taking into account (1.9) and assuming that the necessary conditions of continuity and integrability of functions are satisfied, we obtain

$$I_1 = \int\frac{\partial^2\Omega}{\partial x_k\partial x}(\mathbf{n}\cdot\mathbf{e}_k)\,dS = \int\frac{\partial^2\Omega}{\partial x^2}\,dy\,dz + \frac{\partial^2\Omega}{\partial x\,\partial y}\,dx\,dz + \frac{\partial^2\Omega}{\partial x\,\partial z}\,dx\,dy = \tag{1.14}$$

$$\int\left(-\frac{\partial^2\Omega}{\partial y^2} - \frac{\partial^2\Omega}{\partial z^2}\right)dy\,dz + \int_{z_1}^{z_2}\left(\oint\frac{\partial^2\Omega}{\partial x\,\partial y}\,dx\right)dz + \int_{y_1}^{y_2}\left(\int\frac{\partial^2\Omega}{\partial x\,\partial z}\,dx\right)dy =$$

$$-\int_{z_1}^{z_2}\left(\oint\frac{\partial^2\Omega}{\partial y^2}\,dy\right)dz - \int_{y_1}^{y_2}\left(\oint\frac{\partial^2\Omega}{\partial z^2}\,dz\right)dy = 0$$

$$I_2 = \int\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x}(\mathbf{n}\cdot\mathbf{e}_x)\,dS = \int\left(\frac{\partial T}{\partial x}\right)^2 dy\,dz + \frac{\partial T}{\partial x}\frac{\partial T}{\partial y}\,dx\,dz + \tag{1.15}$$

$$\frac{\partial T}{\partial x}\frac{\partial T}{\partial z}\,dx\,dy = \int (\nabla T)^2\,dy\,dz - \int \left[\left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 \right]\,dy\,dz +$$

$$\int \left(\frac{\partial}{\partial x}\,T\,\frac{\partial T}{\partial y} - T_w\,\frac{\partial^2 T}{\partial x\,\partial y}\right)\,dx\,dz + \int \left(\frac{\partial}{\partial x}\,T\,\frac{\partial T}{\partial z} - T_w\,\frac{\partial^2 T}{\partial x\,\partial z}\right)\,dx\,dy$$

We similarly prove that all integrals in (1.15), with the exception of the first one, i.e.

$$I_2 = \int (\nabla T)^2\,(\mathbf{n}\cdot\mathbf{e}_x)\,dS \tag{1.16}$$

are, as in (1.14), zero. Substituting (1.14) and (1.16) into (1.13), we obtain the sought formula (1.11). Now

$$M_x^{(T)} = -\int [y p_{zk}^{(T)} - z p_{yk}^{(T)}]\,(\mathbf{n}\cdot\mathbf{e}_k)\,dS \tag{1.17}$$

By carrying out operations similar to those used in the derivation of (1.11) and integrating by parts, we obtain (1.12). Note that for Maxwellian molecules $\alpha_2 = 0$, hence in that case local thermal stresses do not affect the force and the moment. We emphasize that Lemma 1 is valid for any finite system of bodies.

In what follows the term single body will be understood to denote a body of finite dimensions in an infinite volume of gas free of any other bodies.

Lemma 2. Let in the gas surrounding the single body

$$\partial P_{ij} / \partial x_j = 0; \quad P_{ij} = o\,(r^{-2}), \quad r \to \infty \tag{1.18}$$

Then $\mathbf{F} = 0$.

In fact, by the Gauss-Ostrogradskii theorem we have

$$F_x = -\int P_{xj}\,(\mathbf{n}\cdot\mathbf{e}_j)\,dS = \int_V \frac{\partial P_{xj}}{\partial x_j}\,dV - \int_\Sigma P_{xj}\,(\mathbf{n}\cdot\mathbf{e}_j)\,d\Sigma$$

where Σ denotes a spherical surface of radius R. Here, by the first condition (1.18) the first integral is zero, while the second by virtue of the second condition (1.18) tends to zero when $R \to \infty$. Since the x-axis is arbitrary, $\mathbf{F} = 0$.

2. First approximation by τ_*.

Let $\tau_* \ll 1$. If there is no stream flowing around the body and $T_w = \mathrm{const}$, a thermal-stress convection induced by thermal stresses [1 – 3] at $v \sim \tau_*^3$, and pressure change $p \sim \tau_*^2$ will generally develop. This means that in the first approximation by τ_* the gas is at rest and $p = 0$. If the motion is induced by other factors, the characteristic velocity is different. Namely, $v \sim \tau_*$. If $v_\infty \sim \tau_*$, or the temperature drop at the body is of the order of τ_* (weak thermal creep), the Barnett terms in the equation of momentum are negligibly small. However in all these cases thermal stresses are quantities of the fundamental order of magnitude, hence

$$p_{ij}^{(T)} = \alpha_1 \eta\,\frac{\partial^2 T}{\partial x_i\,\partial x_j} \sim \tau_* \tag{2.1}$$

and

$$\Delta T = 0 \tag{2.2}$$

which means that they could contribute to $\mathbf{F}$ and $\mathbf{M}$.

However in the first approximation by τ_*, $\mathbf{F}^{(T)}$ and $\mathbf{M}^{(T)}$ are zero. This follows from Lemma 1, if one takes into account that in the considered approximation condition (1.9) at surface S is replaced throughout the gas by Eq. (2.2).

Thus for $T_w = \mathrm{const}$ the force and the moment of forces acting on a separate body of finite dimensions, taken out of a finite system of such bodies, are zero in the first

approximation by τ_*. This statement may be called the generalized Maxwell theorem.

The problem of thermal-stress effects in a gas at rest was first considered by Maxwell. A statement analogous to the one proved here about the force is contained in [6]. Epstein has proved it for a single body (*). This constraint was necessary for using the Gauss-Ostrogradskii theorem and passing to the examination of the asymptotics of p_{ij}^T for $r \to \infty$. Here the proof is simplified, extended to the moment of forces, and to the case of a system of bodies.

3. Electrostatic analogy. When $T_w = $ const , then in the first approximation by τ_* the gas is at rest, but

$$p = -X_T (\nabla \tau)^2, \qquad \tau = T - 1 \qquad (3.1)$$

with relative error τ_*. For τ we then have

$$\triangle \tau = 0, \quad \tau \big|_{S_i} = \tau_i, \quad \tau \, (r \to \infty) \to 0 \qquad (3.2)$$

where S_i is the surface of the i-th body of a finite system of bodies.

Formula (3.1) follows from relationships (1.5) and (1.6). Allowing for (3.1) and (1.11), we obtain

$$F_x = k_T \int\limits_S (\nabla \tau)^2 \, (\mathbf{n} \cdot \mathbf{e}_x) \, dS, \quad k_T = X_{Tw} - \frac{2}{3} \, a_{2w} \qquad (3.3)$$

and a similar expression for M_x. For $\mu = \eta = T^s$ we have $k_T = {}^1/_2 \, (s\omega_1 + \omega_3) > 0$.

It is important to note that relationships (3.2) conform to those for the potential of an electrostatic field outside of conductors in vacuum, and that (3.3) is proportional to the ponderomotive force acting on a conductor in an electrostatic field (see, e. g. [8]).

The electrostatic analogy has thus been established: the forces of interaction between uniformly heated (cooled) bodies with $\tau_i \neq 0$ can be calculated by conventional methods of electrostatics.

The force acting between two spherical particles at relative temperatures τ_1 and τ_2 is asymptotically $(R \to \infty)$ equal

$$F = k_T 8\pi \tau_1 \tau_2 r_1 r_2 R^{-2}$$

where r_1 and r_2 are the radii of particles and R is the distance between these. If sign $\tau_1 = $ sign τ_2, the particles repel each other, and if sign $\tau_1 = -$sign τ_2, they are attracted to each other (Coulomb's law in electrostatics). In particular, a body whose temperature differs from that of a plane screen will be deposited on the screen.

In the τ_*^2 approximation no force is acting on the body. This follows from the analogy, as well as from Lemma 2 and relationships (3.2).

4. The thermophoretic force. Let us consider one of the problems in the approximation described in Sect. 3. Let a uniformly heated spherical particle of radius $L = 1$ be located in a boundless volume of gas. We set $T_* = T_w$, define τ by formula $T = 1 + \iota$, and assume that along the polar axis the temperature gradient at the body $\tau \, (r = 1) = 0$ is specified by

---

*) The authors were unaware of [7] and in the paper [1] had given a proof which repeated that of Epstein's. They have, however, proved a similar statement with respect to the moment of forces, which was not given in [7].

$$\tau \ (r \to \infty) \to Br \cos \theta \ + D \tag{4.1}$$

where B and $D \sim \tau_* \ll 1$. The solution of equation $\Delta\tau = 0$ is

$$\tau = B \ (r - r^2) \cos \theta \ + D \ (1 - r^{-1}) \tag{4.2}$$

which means that at the body $\nabla\tau = 3B \cos \theta \ + D$ and that (3.3) assumes the form

$$F_x = k_T 8\pi BD \tag{4.3}$$

The force acting on the body is independent of the size of the latter and is nonzero if $D \neq 0$ (if $T_\infty \neq 1$ for $B = 0$). When the body is heated $(D < 0)$, the force acts in the opposite direction to the specified ∇T, if it is cooled $(D > 0)$, its direction is the same as ∇T.

Let $C_F = F_x \ (\mu_* \ / \ L\rho_* a_*)^2$, where a_* is the speed of sound. It follows then from (4.3) that

$$C_F \sim \mathrm{Kn}^2 \tau_*^2 \tag{4.4}$$

The phenomenon considered here is a new variant of thermophoresis induced in the gas by thermal stresses for a small but fixed difference between the local temperature of gas and body for the Knudsen number $\mathrm{Kn} \to 0$.

Classical thermophoresis occurs when the thermal conductivity coefficient η_b of the particle is finite. Then in the presence of $\nabla T |_\infty$ the temperature $T_w \neq$ const the thermal creep takes place along the wall, which induces the motion of gas. The related value

$$C_F \sim -\mathrm{Kn}^2 B\eta \ / \ \eta_b \sim -\mathrm{Kn}^2 \tau_* \eta \ / \ \eta_b \tag{4.5}$$

For metal particles $\eta_b \gg \eta$ and, if the particle is superheated or supercooled, (4.5) can be even smaller than (4.4).

Another limit case (Kn small but fixed, $\tau_* \to 0$, and $T_w =$ const) was considered in [9], where thermal stresses were neglected in equations, but were taken into account in boundary conditions for second order creep together with the second order creep induced by the temperature jump. The second effect is sometimes taken into account in the analysis, although it was shown in [9] that the first effect is the determining one, since it defines the sign of the force

$$C_F \sim \mathrm{Kn}^3 B \sim \mathrm{Kn}^3 \tau_* \tag{4.6}$$

The same result was obtained in [10] for a perfectly heat-conducting body.

It follows from formulas (4.4) − (4.6) that, depending on external conditions and properties of particles, various forms of thermophoresis can predominate. In sets of heated (cooled) particles the effect described in Sect. 3 may be the predominant one.

5. Conditions for quiescence around a lightly deformed sphere.

It was shown in Sect. 3 that in the τ_*^2 approximation the force acting on a single body is zero. For arbitrary τ_* the problem of this force remains unresolved. Attempts at establishing whether such force generally exists proved to be unsuccessful. It seemed reasonable to consider simple limit cases. The first of these $(\tau_* \ll 1)$ was considered above. Let us now pass to the case of a slightly deformed uniformly heated sphere and an arbitrary τ_* . As previously, we assume that expansions in a small parameter are admissible, that all derived systems of equations have unique solutions, and that all functions have the necessary properties of smoothness.

Let the relative deformation ε of the sphere be small. We seek the solution in a spherical system of coordinates of the form

$$T = T_0 (r) + \varepsilon T_1 (r, \theta, \varphi) + \dots, \quad \mathbf{v} = \varepsilon \mathbf{v}_1 (r, \theta, \varphi) + \varepsilon^2 \mathbf{v}_2 (r, \theta, \varphi) + \dots$$
$$\Pi = \Pi_0 (r) + \varepsilon \Pi_1 (r, \theta, \varphi) + \dots, \quad F = \varepsilon F_1 + \varepsilon^2 F_2 + \dots$$

where T_0 and Π_0 are solutions of the problem of uniformly heated sphere $(\varepsilon = 0)$ in a gas at rest, T_0 is the solution of the Laplace equation (1.9), thermal stresses are balanced by pressure, and any force is absent [1, 2]. The aim in this case is to show that not for all forms of deformation $\mathbf{v}_1 \neq 0$.

Let us prove that $\mathbf{v}_1 \equiv 0$, then and only then, when

$$T_1 = \eta_0^{-1} [T_{10}^{(0)} (r) + r^{-2} Y_1 (\theta, \varphi)], \quad \eta_0 = \eta (T_0) \tag{5.1}$$

where $T_{10}^{(0)}$ is an arbitrary function of r and $Y_1 (\theta, \varphi)$ is the first spherical-surface harmonic. We call this statement Lemma 3.

It was shown in [1] that the condition of quiescence around a uniformly heated body is

$$\frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_k} \frac{\partial^2 T}{\partial x_k \partial x_j} - \frac{\partial T}{\partial x_j} \frac{\partial T}{\partial x_k} \frac{\partial^2 T}{\partial x_k \partial x_i} = 0 \tag{5.2}$$

In the case of simply connected bodies of finite dimensions this condition is only satisfied for a sphere. It is obtained by cross differentiation of Eq. (1.5) for $\mathbf{v} \equiv 0$. It can obviously be satisfied also in the case of other bodies, if quantities of the order of $O(\varepsilon^2)$ and higher are neglected. Then $\mathbf{v} \leqslant O(\varepsilon^2)$.

The condition of quiescence in a linear approximation by ε is defined as follows:

$$\frac{\partial T_0}{\partial x_i} \frac{\partial T_0}{\partial x_k} \frac{\partial^2 T_1}{\partial x_k \partial x_j} + \frac{\partial T_0}{\partial x_i} \frac{\partial T_1}{\partial x_k} \frac{\partial^2 T_0}{\partial x_k \partial x_j} + \frac{\partial T_1}{\partial x_i} \frac{\partial T_0}{\partial x_k} \frac{\partial^2 T_0}{\partial x_k \partial x_j} - \tag{5.3}$$
$$\frac{\partial T_0}{\partial x_j} \frac{\partial T_0}{\partial x_k} \frac{\partial^2 T_1}{\partial x_k \partial x_i} - \frac{\partial T_0}{\partial x_j} \frac{\partial T_1}{\partial x_k} \frac{\partial^2 T_0}{\partial x_k \partial x_i} - \frac{\partial T_1}{\partial x_j} \frac{\partial T_0}{\partial x_k} \frac{\partial^2 T_0}{\partial x_k \partial x_i} = 0$$

Using here the relationship $x_k \partial / \partial x_k = r \partial / \partial r$

$$T_0 = T_0 (r), \quad \eta_0 T_0'' + \eta_0^{\cdot} (T_0')^2 = - (2 / r) \eta_0 T_0' \tag{5.4}$$

introducing the notation $\quad T_{10} = \eta_0 T_1, \quad (\ )' = d(\ ) / dr \tag{5.5}$

$$Q_{ij} (u) = x_i \frac{\partial u}{\partial x_j} - x_j \frac{\partial u}{\partial x_i}$$

and, as can be readily verified,

$$x_k \frac{\partial}{\partial x_k} Q_{ij} (T) = x_k \left(x_i \frac{\partial^2 T}{\partial x_j \partial x_k} - x_j \frac{\partial^2 T}{\partial x_j \partial x_k} \right) + Q_{ij} (T) \tag{5.6}$$

$$Q_{ij} (\eta_0 T_1) = \eta_0 Q_{ij} (T_1)$$

we reduce (5.3) to the form

$$\frac{\partial}{\partial r} Q_{ij} (T_{10}) + \frac{2}{r} Q_{ij} (T_{10}) = 0 \tag{5.7}$$

Integrating (5.7), we obtain

$$Q_{ij} (T_{10}) \equiv x_i \frac{\partial T_{10}}{\partial x_j} - x_j \frac{\partial T_{10}}{\partial x_i} = f_{ij} (\theta, \varphi) r^{-2} \tag{5.8}$$

where f_{ij} are arbitrary functions of θ and φ. Equalities (5.8) are significant only for $i \neq j$ (for $i = j$ there are no conditions for T_{10}), hence (5.8) reduces to

$$\mathbf{r} \times \bigtriangledown T_{10} = \mathbf{f}\,(\theta,\,\varphi)\,r^{-2} \tag{5.9}$$

or

$$\frac{1}{\sin\theta}\,\frac{\partial T_{10}}{\partial\varphi} = f_1\,(\theta,\,\varphi)\,r^{-2}, \qquad \frac{\partial T_{10}}{\partial\theta} = f_2\,(\theta,\,\varphi)\,r^{-2} \tag{5.10}$$

It follows from (5.10) that

$$T_{10} = T_{10}{}^{(0)}\,(r) + r^{-2} f\,(\theta,\,\varphi) \tag{5.11}$$

For $\mathbf{v}_1 \equiv 0$ the equation of energy for T_{10} assumes the form

$$\bigtriangleup T_{10} = 0 \tag{5.12}$$

The first term in (5.11) belongs to the class of centrally-symmetric temperature fields which do not generate motion and are, consequently, of no interest. The second term assumes by virtue of (5.12) the form (5.1). Thus, if the perturbed temperature field is of the form (5.1), the linearized conditions of quiescence are satisfied and thermal stresses are balanced by pressure. In the opposite case a motion of velocities of the order of ε is generated. The lemma is proved.

The temperature field (5.1) obtains when the sphere deformation is specified by the first spherical harmonic, which in the linear approximation is equivalent to a shift of the sphere in a certain direction.

6. On the force acting on a deformed sphere. Let us prove that the force acting on a slightly deformed ($\varepsilon \ll 1$) and uniformly heated ($\tau_* \sim 1$) sphere is $\mathbf{F} \leqslant O\,(\varepsilon^2)$.

It is necessary to prove that in the linear approximation by ε we have $\mathbf{F} \equiv \mathbf{F}_1 = 0$. We linearize the equations of conservation and write these in the zero and first approximations by ε. omitting the subscript at $\mathbf{v}_1$. We obtain

$$\frac{\partial v_r}{\partial r} + \frac{1}{r}\,\frac{\partial v_\theta}{\partial\theta} + \frac{2v_r}{r} + \frac{v_\theta\,\mathrm{ctg}\,\theta}{r} + \frac{1}{r\sin\theta}\,\frac{\partial v_\varphi}{\partial\varphi} = v_r\,(\ln T_0)' \tag{6.1}$$

$$(r^2\eta_0 T_0')' = 0, \quad \bigtriangleup T_{10} = E v_r\,(\ln T_0)'$$

$$\frac{\partial\Pi_1}{\partial r} = \mu_0\sigma_r + 2\mu_0\dot{}\,T_0'\,\frac{\partial v_r}{\partial r} + 3Y_{T_0}(T_0')^2\,\frac{\partial T_1'}{\partial r} + Y_{T_0}\dot{}\,(T_0')^3 T_1 + E\left(\alpha_1\dot{} - \frac{\alpha_2}{\eta}\right)_0 (T_0')^2\,\frac{v_r}{T_0}$$

$$\frac{\partial\Pi_1}{\partial\theta} = r\mu_0\sigma_\theta + \mu_0\dot{}\,T_0'\left(r^2\frac{\partial}{\partial r}\frac{v_\theta}{r} + \frac{\partial v_r}{\partial\theta}\right) + \eta_0^{-1}Y_{T_0}(T_0')^2\,\frac{\partial T_{10}}{\partial\theta}$$

$$\frac{\partial\Pi_1}{\partial\varphi} = r\sin\theta\mu_0\sigma_\varphi + \mu_0\dot{}\,T_0'\left(r^2\sin\theta\frac{\partial}{\partial r}\frac{v_\varphi}{r} + \frac{\partial v_r}{\partial\varphi}\right) + \frac{Y_{T_0}}{\eta_0}(T_0')^2\,\frac{\partial T_{10}}{\partial\varphi}$$

$$\Pi_1 = p_1 + {}^2\!/{}_3\,(E\alpha_1 + \mu)_0\,v_r\,(\ln T_0)' + 2X_{T_0}T_0'\,\frac{\partial T_1}{\partial r} + X\dot{}_{T_0}(T_0')^2\,T_1$$

where the prime denotes differentiation with respect to r ; μ_0, etc. are functions of T_0, and σ_r, σ_θ and σ_φ denote the related terms of the Navier-Stokes operator $\Pi^{(1)}$, which for $\mu = 1$ can be found in texts on hydrodynamics (see [11]).

Let us establish the boundary conditions for T_1. Let the shape of the body be specified by the expression

$$r = 1 + \varepsilon a\,(\theta,\,\varphi) \tag{6.2}$$

Then at the wall

$$T = T_0 (r, \theta, \varphi) + \varepsilon T_1 (r, \theta, \varphi) + \ldots =$$
$$T_0 (1, \theta, \varphi) + \varepsilon \left[a (\theta, \varphi) \partial T_0 / \partial r + T_1 (r, \theta, \varphi) \right]_{r=1} + \ldots = T_w$$

From which

$$T_0 (1) = T_w, \quad T_1 (1, \theta, \varphi) = - a (\theta, \varphi) T_0' (1) \tag{6.3}$$

Let the perturbation function be of the form

$$a = \sum_{n=1}^{\infty} a_n P_n (\cos \theta) + \sum_{m \leqslant n}^{\infty} Y_{n, m} (\theta, \varphi) \tag{6.4}$$

$$Y_{n, m} = a_{n, m}^{(0)} P_n^{m} (\cos \theta) \cos m\varphi + a_{n, m}^{(1)} P_n^{m} (\cos \theta) \sin m\varphi$$

If the solution of the system of Eqs. (6.1) is sought in the form

$$\eta_0 T_{10} = \Sigma (T_1) + \sum_{n=1}^{\infty} t_n (r) P_n (\cos \theta), \qquad \Pi_1 = \Sigma (\Pi_1) + \sum_{n=1}^{\infty} h_n (r) P_n (\cos \theta) \tag{6.5}$$

$$v_r = \Sigma (v_r) + \sum_{n=1}^{\infty} f_n (r) P_n (\cos \theta), \quad v_\theta = \Sigma (v_\theta) + \sum_{n=1}^{\infty} g_n (r) P_n^1 (\cos \theta), \quad v_\varphi = \Sigma (v_\varphi)$$

where

$$\Sigma (T_1) = \sum_{m=1}^{\infty} [T_1^{(m, 0)} (r, \theta) \cos m\varphi + T_1^{(m, 1)} (r, \theta) \sin m\varphi]$$

with boundary conditions for $r = 1$

$$v_\alpha^{(m, \beta)} = 0 \qquad (\alpha = r, \theta, \varphi; \beta = 0, 1) \tag{6.6}$$

$$T_1^{(m, 0)} = - \eta_0 T_0' \sum_{n=m}^{\infty} a_{n, m}^{(0)} P_n^{m} (\cos \theta)$$

$$T_1^{(m, 1)} = - \eta_0 T_0' \sum_{n=m}^{\infty} a_{n, m}^{(1)} P_n^{m} (\cos \theta)$$

$$f_n = 0, \quad g_n = 0, \quad t_n = - (\eta_0 T_0')_{r=1} a_n \tag{6.7}$$

and zero boundary conditions for $r \to \infty$, then the system of equations for variables with different subscripts are independent of each other and, consequently, the m-th deformation gives rise to the m-th component in T_1, v and Π_1 (because of their unwieldiness these systems of equations are not reproduced here).

Taking into consideration Lemma 1, we write the part of the expression for arbitrarily directed force induced by the perturbed parameters of gas, in a polar system of coordinates whose axis coincides with the direction of the force. We have

$$F_1^{(1)} = - \int_0^\pi \int_0^{2\pi} (P_{rr} \cos \theta - P_{r\theta} \sin \theta)_{r=1} \sin \theta \, d\theta \, d\varphi = \tag{6.8}$$

$$- \int_0^\pi \int_0^{2\pi} \left\{ \left[\Pi_1 - 2 \chi_{T_0} T_0' \frac{\partial T_1}{\partial r} - \chi_{T_0}^{\cdot} (T_0')^2 T_1 - 2\mu_0 \frac{\partial v_r}{\partial r} + \right. \right.$$

$$\left. \frac{4}{3} \alpha_{20} T_0' \frac{\partial T_1}{\partial r} \right] P_1 (\cos \theta) + \mu_0 \frac{\partial v_\theta}{\partial r} P_1^1 (\cos \theta) \right\}_{r=1} \sin \theta \, d\theta \, d\varphi$$

Owing to the orthogonality of the system of functions $\{\sin m\varphi, \cos m\varphi\}$ for $m \geq 1$ with respect to unity in the interval $(0, 2\pi)$. only components with $m = 0$, i. e. f_n, g_n, h_n and t_n, can contribute to $F_1^{(1)}$. It appears that here the equations for groups of variables with different subscripts n are, also, independent of each other, namely

$$f_n' + 2r^{-1}f_n - n(n+1)r^{-1}g_n = f_n(\ln T_0)' \tag{6.9}$$

$$t_n'' + 2r^{-1}t_n' - n(n+1)r^{-2}t_n = Ef_n(\ln T_0)'$$

$$h_n' = \mu_0 [f_n'' + 2r^{-1}f_n' - 2r^{-2}f_n - n(n+1)r^{-2}f_n + 2n(n+1)r^{-2}g_n] +$$

$$2\mu_0^\cdot T_0'f_n' + E\left(\alpha_1^\cdot - \frac{\alpha_2}{\eta}\right)_0 (T_0')^2 \frac{f_n}{T_0} + 3Y_{T0}(T_0')^2 \left(\frac{t_n}{\eta_0}\right)' + Y_{T0}^\cdot (T_0')^3 \frac{t_n}{\eta_0}$$

$$h_n = \mu_0 [rg_n'' + 2g_n' - n(n+1)r^{-1}g_n + 2r^{-1}f_n] +$$

$$\mu_0^\cdot T_0'[r^2(g_n/r)' + f_n] + h_0^{-1}Y_{T0}(T_0')^2 t_n$$

Use was made here of the fact that the deformation for $m = 0$ is axisymmetric (consequently $v_\varphi = 0$), and of the known relationship between Legendre polynomials $P_n(\cos \theta)$ and the first order adjoint Legendre polynomials $P_n^1(\cos \theta)$.

Reverting to (6.8) we see that owing to the orthogonality only terms with subscript $n = 1$, i. e. those due to the first zonal harmonic of stress, can contribute to $F_1^{(1)}$. For $n = 1$ and with allowance for

$$T_0'' = -(\eta_0^\cdot/\eta_0)(T_0')^2 - (2/r)T_0'$$

equations (6.9) with boundary conditions (6.7) and zero boundary conditions for $r \to \infty$ have the solution

$$f_1 = 0, \quad g_1 = 0, \quad t_1 = cr^{-2}, \quad h_1 = Y_{T0}\eta_0^{-1}(T_0')^2 t_1, \quad c = -a_1(\eta_0 T_0')_{r=1}$$

with

$$T_{10} = cr^{-2}\cos\theta \tag{6.10}$$

These results are evidently in accordance with Lemma 3, i. e. the temperature field (6.10) does not unduce motion of the gas. According to Lemma 2 deformations and thermal stresses dependent on the first zonal harmonic are balanced by pressure and the force acting on the body is $F_1 = 0$. The latter can be verified by direct integration over the body and taking into account besides (6.8) the input of the zero approximation of the solution at the deformed surface.

It has thus been shown that, if the deformation is such that $n \geq 2$, then $v_1 \neq 0$ but $F_1 = 0$. If, however, $n = 1$, then $v_1 = 0$, thermal stresses are balanced by pressure, and again $F_1 = 0$. The statement formulated above is proved.

For a single body the results obtained here and in Sect. 3 tend to refute the existence of a thermal-stress force acting on a single body.

7. Stress concentration phenomena (*). Let us consider now similar phenomena produced by concentration stresses in a binary mixture of gases at $n = $ const and $T = $ const (hence the binary diffusion coefficient $D_{12} = $ const). We disregard here the Dufour effect which is absent in the case of Maxwellian molecules and generally results in a weak effect of concentration gradients on T. The reverse effect of temperature gradients on concentration fields and velocities is, obviously, even

---

*) These could be called diffusion phenomena, since such stresses are defined in terms of the "Navier-Stokes" diffusion rates and of their derivatives and, also, by analogy to such concepts as diffusion slip and thermal effects of diffusion.

weaker. Let us consider the first constituent of a mixture as the "basic" one and assume that the characteristic values $\mu_* \equiv \mu_{11}$ and $\rho_* \equiv nm_1$, where μ_{11} is the viscosity coefficient of a pure gas consisting of molecules of mass m_1 of the first constituent.

In the case of slow flows concentration stresses $p_{ij}^{(c)}$ are of the fundamental order of magnitude, if the characteristic differences of concentration are of the order of unity [2]. In a more general form than in [1, 2] this condition may be defined by

$$\sum_\alpha |\nabla y_\alpha| \sim 1, \quad y_\alpha = \frac{n_\alpha}{n}, \quad n = \sum_\alpha n_\alpha \tag{7.1}$$

where summation is carried out over all constituents of the mixture and n_α is the number of particles of the α-kind in a unit of volume. The expression for $p_{ij}^{(c)}$ was obtained in [12] from the equations of moments for a binary mixture of Maxwellian molecules by using Maxwell's method. It conforms to the results obtained by the Chapman-Enskog method in the first approximation by Sonin's polynomials in [13]. Recalling that $y_1 + y_2 = 1$, we write that expression thus

$$p_{ij}^{(c)} = \omega \left[\frac{\partial^2 y_1}{\partial x_i \partial x_j} \right] + \psi \left[\frac{\partial y_1}{\partial x_i} \frac{\partial y_1}{\partial x_j} \right] \tag{7.2}$$

$$\omega = \omega_0 (\Delta \rho^*)^{-1} \{ M_2 - M_1 + \gamma [2(M_1 y_1 - M_2 y_2) + \tag{7.3}$$
$$y_2 S_2 - y_1 S_1] \}$$

$$\psi = \chi - (\omega / \rho^*)(M_1 - M_2)$$

$$\chi = \omega_0 \gamma (\Delta \rho^*)^{-1} [(1 - \gamma)(S_1 M_2 + S_2 M_1) - 1 + \gamma S_1 S_2]$$

$$\omega_0 = 2\gamma (s_{11}^{22} / s_{12}^{22})^2, \quad \rho^* = M_1 y_1 + M_2 y_2, \quad M_\alpha = m_\alpha / (m_1 + m_2)$$

$$\Delta = y_1^2 S_1 (M_2 + M_1 \gamma) + y_2^2 S_2 (M_1 + M_2 \gamma) + y_1 y_2 (1 + \gamma S_1 S_2)$$

$$\mu_{\alpha\alpha} = \frac{5kT}{8\Omega_{\alpha\alpha}^{22}}, \quad \gamma = \frac{3}{10} \frac{\Omega_{12}^{22}}{\Omega_{12}^{11}}, \quad S_1 = (2M_2)^{-1/2} \frac{s_{11}^{22}}{s_{12}^{22}}, \quad S_2 = (2M_1)^{-1/2} \frac{s_{22}^{22}}{s_{12}^{22}}$$

$$s_{\alpha\alpha}^{22} = {}^1/_2 (m_\alpha / \pi k T)^{1/2} \Omega_{\alpha\alpha}^{22}, \quad s_{12}^{22} = {}^1/_2 (2m_1 M_2 / \pi k T)^{1/2} \Omega_{12}^{22}$$

The quantities Ω and s have the same meaning as in [5] and the coefficient γ varies from 0.6 (elastic spheres) to 0.775 (Maxwellian molecules). For molecules considered to be elastic spheres and for Maxwellian molecules the ratio $s_{\alpha\alpha}^{22}/s_{12}^{22}$ is, respectively, $4\sigma_\alpha^2 (\sigma_1 + \sigma_2)^{-2}$ and $(\varkappa_{\alpha\alpha} / \varkappa_{12})^{1/2}$, where $\alpha = 1, 2$, σ_α are the diameters of spheres, and $\varkappa_{\alpha\beta}$ are coefficients in formulas for intermolecular forces.

The equations of conservation can be reduced to the form

$$\nabla v = -v \cdot \nabla \ln \rho^* \tag{7.4}$$

$$\nabla^2 y_1 = [(M_1 - M_2) / \rho^*] (\nabla y_1)^2 + (2\gamma)^{-1} (v \cdot \nabla y_1) \tag{7.5}$$

$$(y_1 + y_2 M_2 / M_1)(v \cdot \nabla) v + \nabla \Pi_c = \Pi^{(1)} + Y_c (\nabla y_1)^2 \nabla y_1 + \tag{7.6}$$
$$(2\gamma)^{-1} (\omega^{\cdot} - \psi)(v \cdot \nabla y_1) \nabla y_1$$

$$\Pi_c = p + X_c \, (\nabla y_1)^2 + [(3\gamma)^{-1}\omega - {}^2/_3\mu] \, (\mathbf{v} \cdot \nabla y_1)$$
$$X_c = {}^1/_2\omega^{\cdot} + {}^1/_6 \psi + 2 \, (3\rho^*)^{-1}\omega \, (M_1 - M_2), \quad A^{\cdot} = dA \, / \, dy_1$$
$$Y_c = {}^1/_2\omega^{\cdot\cdot} - {}^1/_2\psi^{\cdot\cdot} + (\omega^{\cdot} - \psi) \, (M_1 - M_2) \, / \, \rho^*$$

If variables

$$\Omega_c = \ln R, \quad R = y_1 + M_2 \, / \, (M_1 - M_2) = \rho^* \, / \, (M_1 - M_2)$$

are substituted for y_1, Eqs. (7.4) and (7.5) assume the form

$$\nabla \mathbf{v} = - \mathbf{v} \cdot \nabla \Omega_c, \qquad \nabla^2 \Omega_c = (2\gamma)^{-1} \, (\mathbf{v} \cdot \nabla \Omega_c) \qquad (7.7)$$

In addition we obtain the expression

$$p_{ij}^{(c)} = \omega R \left[\frac{\partial^2 \Omega_c}{\partial x_i \partial x_j} \right] + \chi \left[\frac{\partial R}{\partial x_i} \frac{\partial R}{\partial x_j} \right] \qquad (7.8)$$

which is similar to (1.1).

Boundary conditions for $\mathbf{v}$ are the same as in Sect. 1, except that along S instead of the thermal creep rate we have here the diffusion slip rate $v_\tau \sim \partial y_1 \, / \, \partial x_\tau$. Boundary conditions for concentration can be of various kinds; in what follows we select the simplest, namely that along the body surface concentrations are constant, i.e. $y_{1w} = $ const.

A lemma equivalent to Lemma 1 is valid in this case. In fact, along S instead of (1.9) we have $\Delta \Omega_c = 0$, hence the first term in formula (7.8) does not contribute to force $\mathbf{F}^{(c)}$ acting on the body and

$$F_x^{(c)} = - \frac{2}{3} \chi \int (\nabla y_1)^2 \, (\mathbf{n} \cdot \mathbf{e}_x) \, dS \qquad (7.9)$$

The formula for $M_x^{(c)}$ is derived in a similar manner. Let us list the remaining properties which are similar to those obtained above. For $y_{1w} = $ const any diffusion slip is absent and the motion of gas is induced by concentration stresses (concentration-stress convection). If $\nabla y_\alpha \ll 1$, its rates are $v_c \sim Y_c \, (\nabla y_1)^3$ and $p_c \sim -X_c \, (\nabla y_1)^2$. For equal boundary conditions the ratios are $v_c \, / \, v_T = Y_c \, / \, Y_T$ and $p_c \, / \, p_T = X_c \, / \, X_T$ (*). Results obtained in Sects. 2–6 apply quantitatively to gas mixtures, except that the coefficients in formulas for forces are different. Instead of (3.3) we have

$$F_x = k_c \int [\nabla (y_1 - y_{1\infty})]^2 \, (\mathbf{n} \cdot \mathbf{e}_x) \, dS, \quad k_c = \frac{1}{2} (\omega^{\cdot} - \psi) \qquad (7.10)$$

Because of this it is necessary to analyze the coefficients X_c, Y_c and k_c. Let us carry out this analysis for the limit cases usually considered in investigations of thermal diffusion [14]. We assume that m_1, n, μ_{11} and y_1 are fixed, and retain in expansions such number of series coefficients which would result in "nonvanishing" formulas for Y_c. In assigning signs we have to bear in mind the previously indicated range of γ.

a) Isotopic molecules of similar mass and equal cross sections, i.e.

$$M_1 = {}^1/_2 + \varepsilon_m, \, M_2 = {}^1/_2 - \varepsilon_m, \quad S_1 = (2M_2)^{-1/2}, \, S_2 = (2M_1)^{-1/2} \quad (7.11)$$

Expanding these expressions in terms of $\varepsilon_m \ll 1$ and using the notation

---

*) These properties were established in [12], where in the formula for F_k on p. 117 there is an error: the second term must be preceded by a minus sign.

$\varepsilon_0 = \varepsilon_m / (1 + \gamma)$, we obtain

$$\omega = 4\omega_0\varepsilon_0 \left[\gamma - 2 + {}^{1}/_{2}\varepsilon_0 (y_1 - y_2) (4 + 15\gamma - 13\gamma^2) + \right.$$
$$\left. \varepsilon_0^2 y_1 y_2 (26+25\gamma + 88\gamma^2 - 103\gamma^3) + \text{const } \varepsilon_0^2 + O(\varepsilon_0^3)\right]$$
$$\psi = 2\omega_0 (1 + \gamma) \varepsilon_0^2 \left[5\gamma^2 - 9\gamma + 16 - \varepsilon_0 (y_1 - y_2) \times \right.$$
$$\left. (48 + 75\gamma - 68\gamma^2 + 25\gamma^3) + O(\varepsilon_0^2)\right]$$
$$X_c \approx {}^{5}/_{3}\omega_0\varepsilon_0^2 (\gamma^3 - 10\gamma^2 + 13\gamma - {}^{24}/_{5}) < 0$$
$$Y_c \approx 2\omega_0\varepsilon_0^3 (5\gamma^4 + 55\gamma^3 - 165\gamma^2 + 133\gamma - 36) < 0$$
$$k_c = \omega_0\varepsilon_0^2 (-5\gamma^3 - 22\gamma^2 + 23\gamma - 8) < 0$$

b) Isobaric molecules of the same mass and similar cross sections, i.e.

$$M_1 = M_2 = {}^{1}/_{2}, \quad S_1 = 1 + \varepsilon_1, \quad S_2 = 1 + \varepsilon_2$$

For $\varepsilon_1 \ll 1$ and $\varepsilon_2 \ll 1$ we have

$$\omega = 4\omega_0\gamma (1 + \gamma)^{-1} (y_2\varepsilon_2 - y_1\varepsilon_1) E, \quad \psi = 2\omega_0\gamma (\varepsilon_1 + \varepsilon_2) E$$
$$E = 1 - \varepsilon_1 y_1^2 - \varepsilon_2 y_2^2 - 2\gamma (1 + \gamma)^{-1} (\varepsilon_1 + \varepsilon_2) y_1 y_2 + o(\varepsilon^2)$$
$$X_c \approx \frac{\omega_0\gamma (\gamma - 5)(\varepsilon_1 + \varepsilon_2)}{3(1 + \gamma)}, \quad Y_c \approx \frac{2\omega_0\gamma (\varepsilon_1 + \varepsilon_2)}{(1 + \gamma)^2} \times$$
$$\{(\varepsilon_1 + \varepsilon_2) [5\gamma + \gamma^2 + y_1(7 - 6\gamma - \gamma^2)] - \varepsilon_2 (\gamma^2 + 4\gamma + 7)]\}$$
$$k_c \approx -\omega_0\gamma (3 + \gamma) (1 + \gamma)^{-1} (\varepsilon_1 + \varepsilon_2)$$

The signs of coefficients are determined by the signs of ε_1 and ε_2. If $\varepsilon_1 + \varepsilon_2 > 0$, then $X_c < 0$, $k_c < 0$, and $Y_c > 0$ with the additional condition $\varepsilon_2 < 0$.

c) The masses of molecules vary considerably but their "diameters" are the same, i.e.

$$M_1 = 1 - \varepsilon_3, \quad M_2 = \varepsilon_3, \quad S_1 \approx (2\varepsilon_3)^{-1/2}, \quad S_2 \approx 2^{-1/2}$$

For $\varepsilon_3 \ll 1$ we obtain

$$\omega \approx -\omega_0\sqrt{2} (y_1\delta)^{-1}, \quad \psi \approx \omega_0(\gamma + \sqrt{2}) y_1^{-2}\delta^{-1}$$
$$X_c \approx \frac{\omega_0 [\gamma + (3\sqrt{2} + \gamma)\eta]}{6y_1^2\delta^2} > 0, \quad Y_c \approx \frac{\omega_0\eta}{2y_1^3\delta^3} [\gamma + \sqrt{2} + (\gamma - \sqrt{2})\eta] > 0$$
$$k_c \approx -{}^{1}/_{2}\omega_0 (y_1\delta)^{-2} [\gamma - (\sqrt{2} - \gamma)\eta] < 0 \quad (\delta = 1 + \eta, \ \eta = (\sqrt{2} - 1) y_1)$$

We recall that the coefficients which define thermal-stress convection have fixed signs (at least for $\mu = T^s$) , namely : $X_T > 0$, $Y_T < 0$ and $k_T > 0$. The signs of the similar coefficients X_c, Y_c and k_c in the case of gas mixtures depend, generally speaking, on the ratios of masses and collision cross sections of molecules. The most "stable" is the sign of k_c which in cases considered above is $k_c < 0$, hence the analogy with electrostatics is valid but with the "opposite sign".

REFERENCES

1. Galkin, V.S., Kogan, M.N. and Fridlender, O.G., On certain kinetic effects in flows of continuous media. Izv. Akad. Nauk SSSR, MZhG, № 3, 1970.

2. Galkin, V.S., Kogan, M.N. and Fridlender, O.G., On free convec-
    tion in gas in the absence of external forces. Izv. Akad. Nauk SSSR, MZhG, № 3,
    1971.
3. Galkin, V.S., Fridlender, O.G. and Tsar'kova, G.E., Examples
    of thermal-stress convection. Uch. Zap. TsAGI, № 5, 1972.
4. Galkin, V.S., Kogan, M.N. and Fridlender, O.G., Flow past a
    strongly heated sphere by a gas with low Reynolds numbers. PMM Vol. 36, № 5,
    1972.
5. Chapman, S. and Cowling, T.G., Mathematical Theory of Nonuniform
    Gases. Camb. Univ. Press, 1960.
6. Maxwell, J.C., On stresses in rarefied gases arising from inequalities of tem-
    perature. Phyl. Trans. Roy. Soc., Vol. 170, 1879.
7. Epstein, P.S., Zur Theorie des Radiometers. Z. Für Physik, Vol. 54, p. 537,
    1929.
8. Stratton, J., Theory of Electromagnetism. Gostekhizdat, Moscow, 1948.
9. Sone, Y., Flow induced by thermal stresses in rarefied gas. Phys. Fluids, № 8, 1972.
10. Dwyer, H.A., Thirteen-moment theory of the thermal force on a spherical par-
    ticle. Phys. Fluids, № 5, 1967.
11. Kochin, N.E., Kibel', I.A. and Roze, N.V., Theoretical Hydrome-
    chanics, pt. 2. Gostekhteorizdat, Moscow-Leningrad, 1948.
12. Galkin, V.S., Kogan, M.N. and Fridlender, O.G., On concentra-
    tion stress convection and certain properties of slow flows of gas mixtures. Izv.
    Akad. Nauk SSSR, MZhG, № 2, 1972.
13. Shavaliev, M.Sh., The Barnett approximation in multiconstituent gas mix-
    tures. Collection: Kinetic Theory of Gases and Plasma. SO Akad. Nauk SSSR,
    Novosibirsk, 1971.
14. Waldmann, L., Transport phenomena in gases at average pressure. Collection:
    Thermodynamics of Gases, Mashinostroenie, Moscow, 1970. -

Translated by J.J.D.

CONVECTION IN CHEMICALLY ACTIVE MEDIA

We consider the stability of a horizontal layer of a dissociating liquid or gas
subject to a given vertical temperature gradient. We determine conditions for
the appearance of stationary and oscillating convection corresponding to an
arbitrary time for the establishment of local chemical equilibrium.

Two problems concerning the appearance of convection in a layer of dissoci-
ating fluid with an arbitrary dissociation time were investigated in [1]. In the
first of these problems it was assumed erroneously that unlike the temperature
the degree of dissociation does not depend on the height, although there is